Mathematiko

Diocesan Boys' School Mathematics Club 2018-2019
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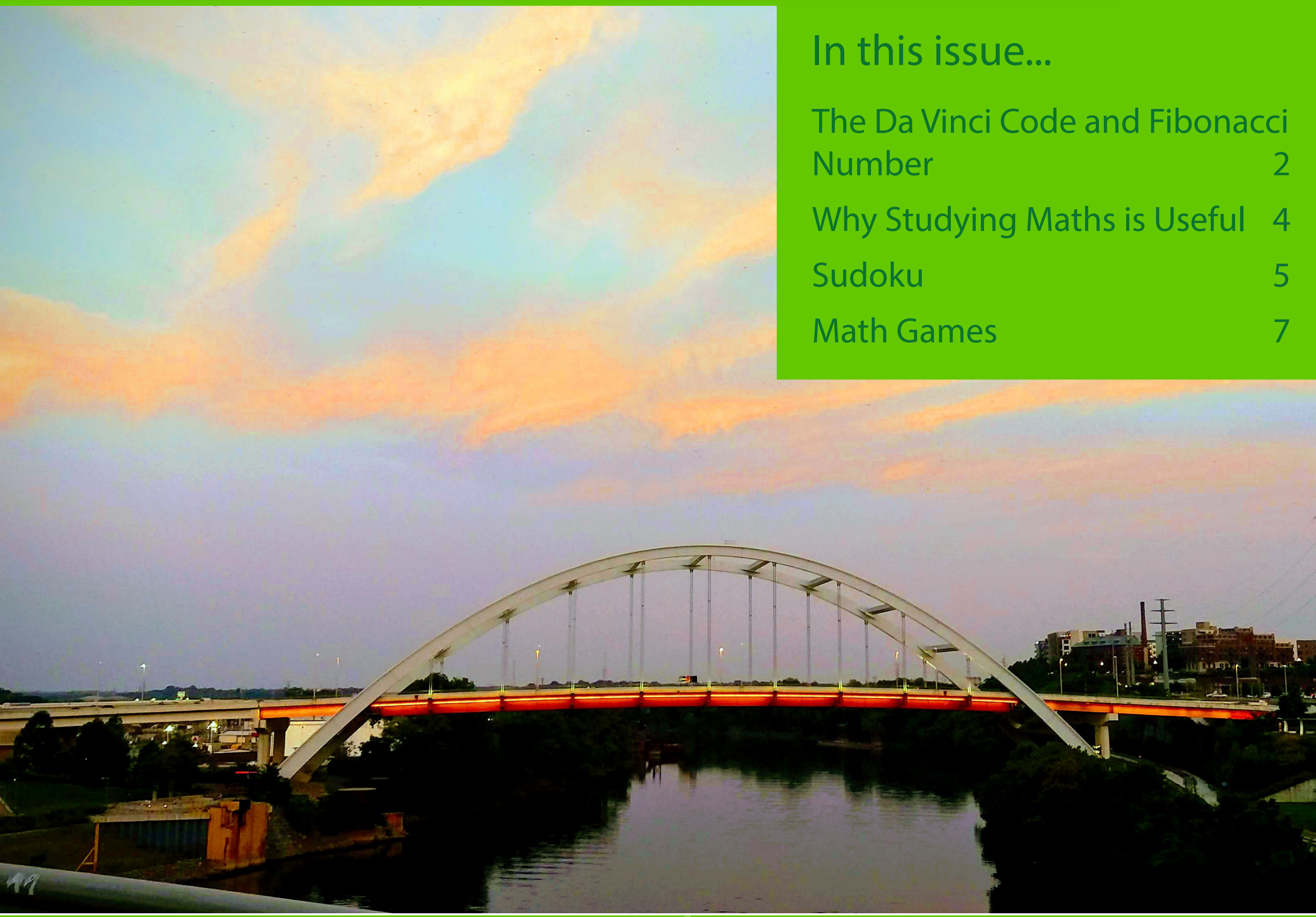
Words From the Editor

Dear readers, it is the time of the year for us to share the joy of Mathematics. Have you ever wondered about the secrets of the Fibonacci numbers or why you should study mathematics? If you had, this issue will be perfect for you! This issue of mathematics includes 2 mathematics articles written by your schoolmate and your teacher. We also included some maths puzzles to entertain you.

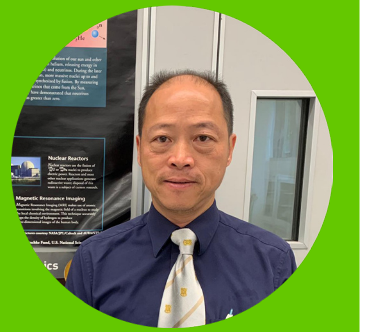
Now turn over the page and begin the journey with us to explore what Maths has to offer for us.

Chan Yuen Ho 10L



In this issue...

The Da Vinci Code & Fibonacci Number



Mr. CY So, IB Mathematics and Physics Teacher

When Dr. Robert Langdon (Tom Hanks) and Sophie Neveu (Audrey Tautou) plugged in the key and were asked to key in a ten-digit account number for a safe box which has not been assessed for twenty years, they were at a loss. They got the key from the clues left by Sophie's grandpa but they did not know the account number. After pondering a while, Dr. Robert Langdon tried 1123581321 and he made it.

This is part of the story of the picture The Da Vinci Code. When I watched Tom Hanks press 1, 1, 2 and 3, I muttered out the remaining digits. My companion heard them and was amazed at what I did. When we left the cinema and sat in a restaurant, she asked me how I knew the digits. I introduced the famous Fibonacci numbers to her :

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, …

Many people know a spreadsheet is a good tool to get the Fibonacci numbers. With the values 1, 1 in cell A1 and A2 respectively and a formula summing up A1 and A2 in A3, one can use the technique copy-and-paste to create up to hundreds of Fibonacci numbers within seconds. However, because of the growing size of the numbers, only the scientific notation form is shown after the 54th Fibonacci number. The last integer form shown is the 54th Fibonacci number 86267571272.

Fewer people know that we can use the Fibonacci numbers to get an approximation of the golden ratio $\frac{1+\sqrt{5}}{2}$. Using the spreadsheet mentioned above, one can first set the formula A2/A1 in B2 and then use the technique copy-and-paste, he can easily get hundreds of the ratios in column B. The ratios seem to converge to 1.618033989.

Part of the spreadsheet which shows the 50th to 54th Fibonacci numbers is shown as below:

	Column A	Column B
50	12586269025	1.618033989
51	20365011074	1.618033989
52	32951280099	1.618033989
53	53316291173	1.618033989
54	86267571272	1.618033989

At this moment, I would like to put forward a question:

Are the values in cells B50 to B54 really equal?

If the answer is yes, then starting from a certain position, Fibonacci numbers form a geometric progression with common ratio equal to the golden ratio.

One may know well those values in B50 to B54 only appear equal but in fact they are not. How do we prove this? One may think about increasing the accuracy of the values in Column B by using other computer techniques. For example, when we calculate the value of π, we use Assembler Language to command the computer to work out as many decimal places of π as possible.

However, no matter how many more digits we set, the values in Column B will finally appear the same as the Fibonacci numbers grow. Thus, we need to use a wiser and simpler way.

Let me focus on the values in A52, A53 and A54 and let them be x, y and z respectively. If the ratios in B53 and B54 are the same, then

$$\frac{y}{x} = \frac{z}{y}$$

$$y^2=xz$$

It is difficult to do the arithmetic as x, y and z are too large. However, as y = 53316291173, the last digit of y^2 is 9. Similarly, the last digit of the product of 32951280099 and 86267571272 is 8. Thus, $y^2 \neq xz$. If follows that the ratios in B53 and B54 are not the same.

What about the rest of the values in column B? How do we prove that none of 3 consecutive Fibonacci numbers form a G.P.? I have thought of a very simple way for it.

At the beginning of my proof, I would like to recall some basic properties of integers.

Rule 1 : The product of an even number and an odd number is even.
Rule 2: The product of two even numbers is even.
Rule 3 : The product of two odd numbers is odd.
Rule 4 : The sum of an even number and an odd number is odd.
Rule 5 : The sum of two odd numbers is even.

Let us have a closer look at the Fibonacci numbers.

The first one is	1	(which is odd)
The second one is	1	(which is odd)
The third one is	2	(which is even)
The 4th one is	3	(which is odd)
The 5th one is	5	(which is odd)
The 6th one is	8	(which is even)
The 7th one is	13	(which is odd)
The 8th one is	21	(which is odd)
The 9th one is	34	(which is even)
…		

It is not difficult to observe the pattern "…, odd, odd, even, …". It is easy to explain the pattern using Rule 4 and Rule 5.

Thus, for any 3 consecutive Fibonacci numbers a, b and c, they must be in one of the following forms:

Form 1 : a is odd, b is odd and c is even
Form 2 : a is odd, b is even and c is odd
Form 3 : a is even, b is odd and c is odd

For Form 1, b^2 is odd since b is odd. However, ac is even as c is even. Thus, $b^2 \neq ac$.

For Form 2, b^2 is even since b is even. However, ac is odd as both a and c are odd. Thus $b^2 \neq ac$.

The argument for Form 3 is just the same as that for Form 1.

Thus, no 3 consecutive Fibonacci numbers form a G.P.. However, as the Fibonacci numbers grow, the ratio of two consecutive Fibonacci numbers tends to be the golden ratio. The proof is beyond the scope of the discussion in this article. Interested readers may refer to the following website for details:

https://en.wikipedia.org/wiki/Fibonacci_number

Mathematics in Daily Life

Mathematics is an important aspect of daily life. We can often see the footprints of Mathematics through everyday items.

For example, the fibonacci numbers appear as the amount of petals of different types of flowers.

While the golden ratio usually appear when investigating architectures. For example, the Parthenon in Greece is a showcase of the golden ratio at work.

You can try to observe the surrounding more often to see if you can find beauty of Mathematics hidden in Mother Nature.

Why Studying Maths is Useful?

Chan Yuen Ho 10L

People often find maths boring and useless as there is little to no correlation between the things you learn in maths class and real life. However, this is not the case. Maths is the foundation of most sciences and is used in many different areas in daily life.

Let's start with a more interesting topic - probability. Probability is to measure how likely an event is going to happen. If an event is impossible to happen, the probability of it is 0. If an event is certainly going to happen, the probability of it is 1. Using maths to calculate the probability and expected value, we could determine the likelihood of something happening and make rational decisions based on the calculated result. Let's play a game, roll a 20-sided die, if the number is larger than 4, you will win the game and get the money equivalent to the number you rolled as a reward after you pay 10 dollars for playing this game. Doing the math will tell you that the expected gain of this game is 0, which means the host nor you will win money in the long run as expected gain for you is 0.

For younger readers, you might not know what is expected value. Expected value is the value you are expected to win or loss after many times of doing an event.

How do you calculate the expected value of an event? You add up the value times the probability of it happening.

Example: You have two 10 HKD bills, two 20 HKD bills and a 100 HKD bill in your wallet.

If you randomly take out a bill from your wallet, the expected value is

P(10) x 10 + P(20) x 20 + P(100) x 100 = 32

With that in mind, let's see why it is a bad idea to buy the Mark 6 with the help of combinatorics and probability. Let's first calculate how many possible ways to win each prize.

First prize (all 6 numbers):
6C6 = 1

Second prize (5 of the 6 numbers + special number)
6C5 × 1C1 = 6

Third prize (5 of the 6 numbers)
6C5 × 42C1 = 258

Forth prize (4 of the 6 numbers + special number)
6C4 × 1C1 × 42C1 = 630

Fifth prize (4 of the 6 numbers)
6C4 × 42C2 = 12915

Sixth prize (3 of the 6 numbers + special number)
6C3 × 1C1 × 42C2 = 17220

Seventh prize (3 of the 6 numbers)
6C3 × 42C3 = 229600

Total possible amount of Mark 6 combinations:
49C6 = 13983816

As the cash prize of Mark 6 for each draw is not the same, I will base the calculation of the expected value on the cash prize value that was given on the draw on PI Day 2019 (also known as 14/3/2019)

Expected value of Mark 6
= (1 × 23215520 + 6 × 305240 + 258 × 73990 + 630 × 9600 + 12915 × 640 + 17220 × 320 + 229600 × 40) ÷ 13983816
= 5.23

Expected gain
= 5.23 - 10
= -4.77

From the math, we can see that the Mark 6 is favorable to the host, and that's why you shouldn't play it as you will lose money in the long run. This is just an example of what probability can tell you about a game. You could try to practise your skills learnt from this article to calculate the expected value of other gambling related games like fish-shrimp-crab, dai-sai. You will probably find that the expected gain is negative as the host of the game needs to win money.

Other than calculating if a game is worth playing, probability can also be applied in many daily life situations. For example, the typhoon route that we go to hko.gov.hk to see everytime there is a typhoon coming to Hong Kong is the line of typhoon path which has the highest probability to be true.

Concept Corner - nCr

nCr is a common notation used in combinatorics. The notation means how many possible combinations are there to choose r objects from n objects.

nCr can be calculated with $\frac{n!}{(n-r)!r!}$

Example: How many ways are there to choose 2 people from 4 people?

Answer: 4C2 = 4 × 3 ÷ 2 = 6

(Extra)

The possible combinations are:

AB, AC, AD, BC, BD, CD

Sudoku

Mathematics Club Board

Though the enthusiasm in playing Sudoku has recently declined, it is still a mathematical game that is worth wide discussion.

Techniques

Basic

Just seek for the missing 1 out of the 9 numbers in each row, column or sub-squares. It is the first and the easiest way for completing any Sudoku, and all of you should know this already.

Intermediate

By scanning 2 or 3 of the column, row and

	1	2	3	4	5	6	7	8	9
1	8	2 7	6	4	9	5	3 7	2 3	1
2	1 2	3	1 2 4	8	7	6	5	9	2 4
3	5	9	4 7	2 3	1 2 3	1 2	8	6	4 7
4	4	5	3	1	6	8	2	7	9
5	1 2	6	9	2 3 7	4	2 7	1 3	5	8
6	7	1 2	8	5	2 3	9	4	1 3	6
7	3	4	2 7	6	1 2	1 2 7	9	8	5
8	6	8	1 2	9	5	3	1 7	4	2 7
9	9	1 2 7	5	2 7	8	4	6	1 2	3

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subsquare at the same time, a few numbers should be easily picked out. By this mean, the game should have been at least half completed.

Advanced

By double implication, some puzzles can even be completed by considering both choices of a square to get another number. Consider the figure on Page 9. The square in row 2 column 1 (21), there are two options, namely 1 and 2. Assume the square contains 2, the square in row 1 column 2 (12) will be 7 due to the cancellation of the choice “2”. Assume the square contains 1, 51 will be 2; 62 will be 1; 68 will be 3, 18 will be 2, and so 12 = 7.

By observing this, you may see that whichever number 21 contains, 12 will be 7. This can give you a helping hand whenever you get stuck in difficult puzzles, but an expert’s technique is required. It's your turn now! Try it below.

	7			5			6	
	8				9	4		
					1			
	1							3
5			6					
		9				8		5
7	4		8					
								7
	6			4			5	

1				4			6	
4					3	1	5	
		3	7			2		
	4							8
		8			2	3		6
		2		3	7			1
	8	4	9	6				
			5			4	1	
	9						8	2

				4				
	3	9	1		7	2	5	
	6		2		9		3	
	1	4	7		6	8	9	
2								6
	9	8	5		3	7	4	
	2		9		4		6	
	4	3	6		2	9	8	
				7				

2								7
	3	5	4	6	7	2	9	
	8	9				1	4	
	1		5	7	6		8	
	6		3		2		7	
	7		8	4	1		6	
	2	1				7	3	
	9	3	7	1	4	8	2	
8								4

Answers can be found on page 8

MAKE 150

Make 150 is a game that the Mathematics Club has created for this year's Garden Fete. The rules are simple, you use any amount of mathematic symbols or operation to make the number 150. To get the numbers, you just need to roll 5 10-sided dice (also known as d10s) or use a set of 50 cards (which has 5 cards for each integer from 1 to 10).

Here are a few sets of number I just generated for you guys to make 150 with mathematics symbol :D

4, 6, 9, 6, 10

8, 4, 1, 1, 5

8, 7, 5, 9, 5

6, 9, 2, 4, 5

9, 6, 10, 1, 10

4, 7, 10, 3, 2

9, 1, 9, 1, 10

9, 7, 6, 5, 6

8, 6, 4, 5, 10

8, 9, 6, 5, 3

Answers can be found on page 8

NONOGRAM

Nonogram, also known as picross, is a game which involves players filling in squares according to the numbers given. The finished product is usually a binary image. The numbers in each column or row represent the number of consecutive colored cells in that column or row. For example, the clue "3 2 3" means that there are sets of 3,2,3 colored cells in that order and has at least one uncolored cell to separate the 3 groups.

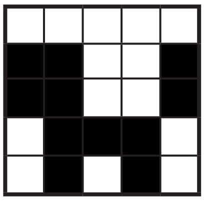
		2	4	1	2	2
	0					
2	1					
2	1					
	3					
1	1					

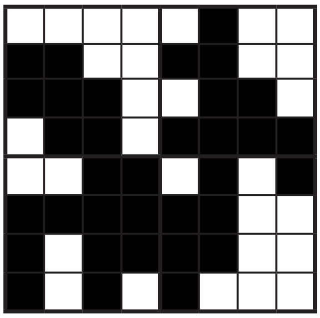
							1			
			2	3			1			
			3	1	6	3	3	7	2	2
		1								
	2	2								
	3	2								
	2	4								
2	1	1								
		6								
	1	4								
1	1	1								

Solutions for Sudoku and Nonogram

1	7	4	2	5	8	3	6	9
2	8	5	3	6	9	4	7	1
3	9	6	4	7	1	5	8	2
4	1	7	5	8	2	6	9	3
5	2	8	6	9	3	7	1	4
6	3	9	7	1	4	8	2	5
7	4	1	8	2	5	9	3	6
8	5	2	9	3	6	1	4	7
9	6	3	1	4	7	2	5	8

1	7	5	2	4	9	8	6	3
4	2	9	6	8	3	1	5	7
8	6	3	7	1	5	2	9	4
3	4	7	1	9	6	5	2	8
9	1	8	4	5	2	3	7	6
6	5	2	8	3	7	9	4	1
2	8	4	9	6	1	7	3	5
7	3	6	5	2	8	4	1	9
5	9	1	3	7	4	6	8	2





1	8	2	3	4	5	6	7	9
4	3	9	1	6	7	2	5	8
5	6	7	2	8	9	1	3	4
3	1	4	7	2	6	8	9	5
2	7	5	4	9	8	3	1	6
6	9	8	5	1	3	7	4	2
8	2	1	9	3	4	5	6	7
7	4	3	6	5	2	9	8	1
9	5	6	8	7	1	4	2	3

2	4	6	1	8	9	3	5	7
1	3	5	4	6	7	2	9	8
7	8	9	2	3	5	1	4	6
3	1	4	5	7	6	9	8	2
5	6	8	3	9	2	4	7	1
9	7	2	8	4	1	5	6	3
4	2	1	6	5	8	7	3	9
6	9	3	7	1	4	8	2	5
8	5	7	9	2	3	6	1	4

Solutions for Make 150 can be downloaded on the website https://bit.ly/DBS_Maths_Club_make_150_solution

However, solutions that require mathematical operations other than add, subtract, multiply and divide will not be shown and may be listed as having "No solution" in the file. For example, the solution for 1, 9, 1, 9, 10 requires the use of square roots.

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Mathematics Club

Founded in 1986, the Mathematics Club continues to enhance students' interest in Mathematics through a series of activities. This year, there was an additional element in our mission - the promotion of Science, Technology, Enginnering and Mathematics (STEM). To fulfil our aims, our club has organized the Garden Fete stall "Make 150" with the education of mathematical operations. There was also a regular publication of out newsletter, Mathematiko. Hollistically, with the assistance of our executive members and teacher-in-charge, and the participation of passionate members, I am confident that next year will be a fruitful one for our club.

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